

Aristotle's Assertoric Syllogistic and Modern Relevance Logic *

Abstract: This paper sets out to evaluate the claim that Aristotle's Assertoric Syllogistic is a relevance logic or shows significant similarities with it. I prepare the grounds for a meaningful comparison by extracting the notion of relevance employed in the most influential work on modern relevance logic, Anderson and Belnap's *Entailment*. This notion is characterized by two conditions imposed on the concept of validity: first, that some meaning content is shared between the premises and the conclusion, and second, that the premises of a proof are actually used to derive the conclusion. Turning to Aristotle's *Prior Analytics*, I argue that there is evidence that Aristotle's Assertoric Syllogistic satisfies both conditions. Moreover, Aristotle at one point explicitly addresses the potential harmfulness of syllogisms with unused premises. Here, I argue that Aristotle's analysis allows for a rejection of such syllogisms on formal grounds established in the foregoing parts of the *Prior Analytics*. In a final section I consider the view that Aristotle distinguished between validity on the one hand and syllogistic validity on the other. Following this line of reasoning, Aristotle's logic might not be a relevance logic, since relevance is part of syllogistic validity and not, as modern relevance logic demands, of general validity. I argue that the reasons to reject this view are more compelling than the reasons to accept it and that we can, cautiously, uphold the result that Aristotle's logic is a relevance logic.

Keywords: Aristotle, Syllogism, Relevance Logic, History of Logic, Logic, Relevance

1. Introduction

The literature on Aristotle's Syllogistic of the past 50 years has frequently suggested that Aristotle's logic bears a similarity to modern relevance logic (Burrell 1964; Smiley 1973, Thom 1981; Smith 1989; Smiley 1994; Gabbay and Woods 2001; Woods 2001; Woods and Irvine 2004; Smith 2007; Striker 2009; Malink 2013).¹ These suggestions range from careful statements like Smith's (2007; cf. 1989, p. 210),

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¹ We could add Normore (1993, p. 448) who claims that '[a]ncient logics were all in some sense relevance logics.'

according to whom some passages ‘could be [...] interpreted as committing Aristotle to something like a relevance logic’, to bolder claims like Smiley’s (1994, p. 30), who thinks that Aristotle is ‘redolent of relevance’, to assertions such as those of Gabbay and Woods, who straightforwardly call Aristotle ‘the first relevant, intuitionistic, non-monotonic, hyperconsistent logician’ (2001, p. 144; cf. Woods and Irvine 2004, p. 66). Given the frequency of this thesis it is surprising to see that there is hardly any attempt to give an argument for it. In those rare cases in which an argument is given, the analysis is confined to only a few lines of the Aristotelian text. There seems to be, therefore, room for a more detailed study of the relation between Aristotle’s syllogistic theory and modern relevance logic.

In order to approach this issue we first have to characterise the notion of relevance in modern relevance logic, as well as, and no less importantly, its place in the modern theory. To do so I will turn to the most influential work on relevance logic to date, A. Anderson and N. Belnap’s 1975 *Entailment*, still considered the *magnum opus* of the field. Relevance, according to these authors, is a question of the validity of arguments, i.e. a question concerning the relation between premises and conclusion. Generally speaking, an argument is valid if the conclusion is a logical consequence of the premises. Classical propositional logic considers relevance between premises and conclusion unimportant for the question whether the conclusion is a logical consequence of the premises: an argument is classically valid if and only if it is impossible for the conclusion to be false while all the premises are true. This is precisely what Anderson and Belnap take issue with: relevance, for them, is integral to the question whether something is a logical consequence of something else, and, hence, relevance is an essential element of the notion of validity. If, therefore, the evaluation of the thesis that Aristotle’s logic is a relevance logic (or

something similar to it) is to turn out positive, it will have to be Aristotle's notion of validity that contains appropriate restrictions on logical consequence.

Anderson and Belnap understand logical relevance in a particular way. The necessity to incorporate relevance into the logical notion of validity, and therefore to incorporate it in a formal way, leads them to suggest two conditions that an argument must satisfy in order to be valid: first, the premises and the conclusion have to *share* some non-logical content (in their case, a propositional variable) – this is understood to be a necessary condition; secondly, all the premises of an argument must be *used* in order to derive the conclusion – this condition is both necessary and sufficient. Since these conditions mark out the notion of relevance in modern relevance logic à la Anderson and Belnap it will be of particular interest whether Aristotle's syllogistic imposes similar restrictions or not.

I should point out two caveats of my paper. First, I will not try to give an answer to the interesting question whether Aristotle had the intention or motivation to incorporate relevance into his syllogistic. It is true that intention and motivation played a role in the historical development of modern relevance logic and it is possible that Aristotle, too, set out to fight the irrelevance between premises and conclusion he encountered in the arguments of his contemporaries; but all that matters for my evaluation of the thesis that Aristotle's logic shows similarity to modern relevance logic is whether his syllogistic contains the aforementioned properties, irrespective of whether relevance was an idea he consciously built into his theory or not.² Secondly, I use the term 'relevance logic' to denote the core of the logic

² The development of modern relevance logic happened against the background of a dominant logical tradition (called 'the Official view' by Anderson and Belnap) that, for Anderson and Belnap, was problematic insofar as it took no account of the notion of relevance and that, in particular, sanctioned as valid the so-called paradoxes of implication. At the same time, Anderson and Belnap considered their intuition that relevance should be a matter of logic to be widely shared: most textbooks on logic, they observe, emphasize the importance of relevance in their informal treatments but advocate classical

developed by Anderson and Belnap. There might be other concepts of logical relevance and different logics that incorporate these concepts, but I will not consider whether other such concepts exist, whether they can be employed in the realm of logic, or how these logics are related to the relevance logic of Anderson and Belnap. I will focus on the question whether Aristotle's assertoric syllogistic is or is not a relevance logic in the sense of Anderson and Belnap and all those who agree with the core convictions put forward by them.

The paper has three parts: in the first, I will turn to Anderson and Belnap's core account of relevance. I will briefly present their reasoning for why relevance has to be part of the notion of validity as well as the two conditions they propose as capturing an appropriate notion of relevance.³ The second part of the paper consists in a detailed textual analysis of those parts of the *Prior Analytics* that, in my view, are most promising for revealing comparable properties of the syllogistic. The third part of the paper discusses an objection against the claim that Aristotle's logic is a relevance logic. This objection emerges from the claim that Aristotle understood syllogistic validity as a subproperty of a more general notion of validity. If this claim is accepted, showing that syllogistic validity incorporates validity is not sufficient for showing that Aristotle's logic is a relevance logic. For, the objection says, syllogistic validity might

validity in their formal treatments; a good example of this is Lemmon (1968). These two factors, historically speaking, provided the impetus for the development of relevance logic. Aristotle, on the other hand, in all likelihood did not work against a dominant logical tradition, at least not against a formalized and generally accepted logic that occupied the position of an 'Official View'. However, he still might have been prompted by the moves of certain Sophists to shield his logic from the dangers of irrelevant premises. Cf. *Topics* VIII.11, in particular 161b24-33 and 162a12-5.

³ I will not provide a defence of Anderson and Belnap's relevance logic. Given the considerable literature on the debate between relevance logicians and advocates of classical as well as of other strains of logic, this is well beyond the scope of this paper.

incorporate relevance, but modern relevance logic demands that relevance be part of the notion of validity itself.

2. The concept of relevance in modern relevance logic

The historical development of modern relevance logic in the 20th century⁴ is not important for the present project and we can base our presentation of the essential elements of modern relevance logic entirely on *Entailment*.⁵ Anderson and Belnap's starting-point is the observation that, although the millennia-old tradition of logic is full of remarks to the effect that relevance between premises and conclusion is of importance, classical validity does not account for this and hence sanctions as valid some arguments the premises of which have no relevance for the conclusion (Anderson and Belnap 1975, p. 17). In order to illustrate this observation, they cite several arguments like this: 'Assume that snow is puce. It follows that (or consequently, or therefore, or it may validly be inferred that) seven is a prime number' (1975, p. 14; cf. pp. 17-18). Their real test cases, however, are the so-called paradoxes of implication.

Among these paradoxes⁶ are, for instance, the following two theorems:

$A \rightarrow (B \rightarrow A)$, and $B \rightarrow (A \rightarrow A)$. In classical propositional logic both can be validly

⁴ Important steps after the advent of the classical propositional calculus due to Frege (1879) and Whitehead and Russell (1910-1913) include MacColl's (1908) criticism of the implication connective, Lewis' (1918) proposal of a 'strict implication' and Ackermann's (1956) 'rigorous implication'.

⁵ Anderson and Belnap's system is not the only one that claims the title of being a relevance logic (cf. Dunn and Restall 2002, pp. 1-2; Priest 2008, pp. 188-220), but since we are only interested in the essential ideas and concepts, what we will extract from *Entailment* will also be true for other members of the family of relevance logic (cf. the criteria for membership proposed by Mares and Meyer (2001, p. 286). -- I should point out that the focus on the pure implicational fragment in my presentation of Anderson and Belnap should not be understood as an affirmation of the view that syllogisms are implications (cf. Łukasiewicz 1957; Corcoran 1974a). The aim of this section is to extract the concept of relevance employed in the logic of Anderson and Belnap and for that the pure implicational fragment is the best choice.

⁶ Whether the so-called paradoxes of implication are real paradoxes or not depends on how we understand 'paradox'. Some authors, understanding 'paradox' as something that leads to logical contradictions, do not think that the paradoxes of implication are real paradoxes because they do not

proven, as can be shown, for instance, by using a system of natural deduction (I follow Anderson and Belnap and use Fitch's system) with rules preserving validity as understood by the classical account⁷:

1	A	hyp	1	B	hyp
2	B	hyp	2	A	hyp
3	A	1 reit	3	A	2 rep
4	B → A	2-3 → I	4	A → A	2-3 → I
5	A → (B → A)	1-4 → I	5	B → (A → A)	1-4 → I

The $\rightarrow I$ rule, used at steps 4 and 5 in both proofs accords with the following deduction theorem: if there exists a proof of B on the hypotheses A_1, \dots, A_n , then there exists a proof of $A_n \rightarrow B$ on the hypotheses A_1, \dots, A_n ; and conversely. It is here, precisely, where Anderson and Belnap think that the classical view goes astray; commenting on the proof of $B \rightarrow (A \rightarrow A)$ they say: 'though our eyes tell us that we proved it [i.e. $A \rightarrow A$, PS] *under* the hypothesis B, it is crashingly obvious that we did not prove it *from* B' (1975, p. 18, italics in the original).

Of course, classical logicians are not confused by the visual order of such proofs – the classical validity of the two theorems cited above can be established by truth tables, too; and Anderson and Belnap do not think that classical logicians are guilty of such a blunder either. The problem, according to Anderson and Belnap, lies much deeper, namely in the account of validity that sanctions the rules used in the proofs above, and in particular in the classical understanding of the implication connective.⁸ Logicians of the classical persuasion are not troubled by the fact that proofs like the above are possible and in general that the so-called paradoxes of

lead to logical contradictions, even though they might regard them as highly counter-intuitive and possibly problematic for that reason; cf. (Sainsbury 2009, pp. 1-2).

⁷ A basic set of rules contains: 1) hypothesis introduction (hyp): hypotheses may be introduced at any time; 2) repetition (rep): a formerly introduced hypothesis may be repeated in the same proof at any time; 3) reiteration (reit): a formerly introduced hypothesis may be repeated in a sub-proof; 4) entailment introduction ($\rightarrow I$): if a proof concludes B from hypothesis A, an implication $A \rightarrow B$ may be introduced; 5) entailment elimination ($\rightarrow E$): given A, B may be deduced from $A \rightarrow B$.

⁸ The implication connective is, for Anderson and Belnap (1975, pp. 3, 5, 12-13), the 'heart of logic', because of its most intimate connection to logical consequence.

implication are valid, for although some might agree that there is a difference between the meaning of ‘implication’ in logic and the meaning of ‘implication’ in natural language, they are satisfied with the fact that implication as understood by classical logic will never lead from truth to falsity. This, however, is not enough for Anderson and Belnap, who call for a stronger relation between antecedent and consequent, a relation much closer to the sense that terms like ‘follow’ and ‘imply’ have in natural languages like English. This relation does not sanction as valid $B \rightarrow (A \rightarrow A)$, i.e. an implication whose antecedent is totally irrelevant to the (although necessarily true) consequent $A \rightarrow A$. This irrelevance becomes immediately obvious when we assign to B ‘the cat sleeps’ and to A ‘the universe is infinite’: ‘the cat sleeps’ implies that ‘if the universe is infinite, then the universe is infinite’ can be validly proven in classical propositional logic, but ‘the cat sleeps’ has no bearing whatsoever on ‘if the universe is infinite then the universe is infinite’.

It is clear, then, that Anderson and Belnap want – and need – to incorporate relevance into the concept of validity, i.e. they are looking for a system ‘for which there is provable a deduction theorem to the effect that there exists a proof of B *from* [as opposed to merely *under* in the sense explicated above, PS] the hypothesis A if and only if $A \rightarrow B$ is provable’ (Anderson and Belnap 1975, p. 19, italics in the original). In order to do so, they propose a notion of relevance characterised by two conditions: *variable-sharing* and *usage*. The principal idea behind the first condition is that relevance requires some meaning content common to both premises and conclusion. The total lack of commonality between the meaning content of the premises and the conclusion and the resulting irrelevance of the premises for the conclusion is illustrated in implications like ‘the cat sleeps’ implies that ‘if the universe is infinite then the universe is infinite’. Since in propositional logic content

is carried by the propositional variables, the first condition of relevance, then, states that in any validly provable implication $A \rightarrow B$, A and B must share a variable (Anderson and Belnap 1975, pp. 32-33).

Variable-sharing is merely a necessary condition. The second condition suggested by Anderson and Belnap, on the other hand, is both necessary and sufficient and it is therefore of greater significance for the understanding of the concept of relevance at work. The second condition, which I will call *usage*, states that all the hypotheses of a proof must be used in deriving the conclusion. The underlying intuition is that ‘for A to be relevant to B it must be possible to *use* A in a deduction of B’. Anderson and Belnap suggest this is plausible because it is a common procedure to check whether the hypotheses in a proof have been used to derive the conclusion (Anderson and Belnap 1975, pp. 30-31). In order to understand how Anderson and Belnap conceive of this condition and to highlight the effect that the condition has for the case of the so-called paradoxes of implication, it is helpful to look at its formal instantiation. Anderson and Belnap propose to supplement the natural deduction calculus with a technique to keep track of the steps in a proof that are actually used; the employment of such a method will rule out those cases in which a conclusion is merely established *under* but not *from* the given hypotheses, or so Anderson and Belnap contend. This technique consists in adding an index number to each hypothesis introduced, charging a consequent with the respective number on entailment elimination ($\rightarrow E$) and discharging it on entailment introduction ($\rightarrow I$).⁹

⁹ More precisely, Anderson and Belnap’s reformed rules of the natural deduction system used before are: ‘(1) one may introduce a new hypothesis $A_{\{k\}}$, where k should be different from all subscripts on hypotheses of proofs to which the new proof is not subordinate; (2) from A_a and $A \rightarrow B_b$ we may infer $B_{a \cup b}$; (3) from a proof of B_a from the hypothesis $A_{\{k\}}$ we may infer $A \rightarrow B_{a - \{k\}}$, provided k is in a; and (4) reit and rep retain subscripts (where a,b,c, range over sets of numerals)’ (1975, p. 22).

With this technique imposed on the natural deduction calculus, the previous deductions of the so-called paradoxes of implication are no longer possible. Consider the following proofs:

1	A _{1}	hyp	1	B _{1}	hyp
2	B _{2}	hyp	2	A _{2}	hyp
3	A _{1}	1 reit	3	A _{2}	2 rep
4	B → A _{?}	2-3 → I*	4	A → A _{}	2-3 → I
5	A → (B → A) _{?}	1-4 → I*	5	B → (A → A) _{?}	1-4 → I*

The left-hand side proof fails at step 4 (and for the same reason at step 5) for the introduction of the entailment $B \rightarrow A$ is not sanctioned by the entailment-introduction rule: you may only introduce an entailment if the antecedent's set of tracking-numerals contains the consequent's set of tracking-numerals, which here is not the case. The right-hand side proof fails for the same reason, but only at step 5; the introduction of the entailment $A \rightarrow A$ is acceptable and the tracking-numerals at this point are completely purged; the next step, then, fails again because of rule (3) stating that k must be in a . The idea that in a valid proof all of the hypotheses have been used has its formal equivalent in the complete disappearance of the tracking numerals (for, the respective numerals get discharged, precisely, on usage); to illustrate this, I have reproduced the proof of the law of assertion that Anderson and Belnap (1975, p. 22) offer:

1	A _{1}	hyp
2	A → B _{2}	hyp
3	A _{1}	1 reit
4	B _{1,2}	2-3 → E
5	(A → B) → B _{1}	2-4 → I
6	A → ((A → B) → B)	1-5 → I

This brief presentation of the core idea of relevance logic à la Anderson and Belnap and its basic implementation can be summarized as follows. Against the supposition that relevance is a concept too vague to be formalized, Anderson and

Belnap propose two conditions to capture this concept: (1) as a necessary condition, the set of premises and the conclusion share at least one variable and hence some meaning content, and (2) as a necessary and sufficient condition, all the premises of a proof are used to deduce the conclusion. Of course, since the second condition is necessary and sufficient, it makes the first condition redundant as far as the enforcement of relevance is concerned: if the second condition is satisfied, the first condition will be satisfied as well. But although the first condition loses its significance for producing the desired effect, it is still important to elucidate the concept of relevance: necessary conditions always help us to understand the things whose conditions they are.¹⁰ Most fundamentally, moreover, Anderson and Belnap argue that (3) relevance is part of the notion of validity.¹¹ I will now turn to Aristotle's investigation of the syllogism in order to determine whether or not these three conditions are, completely or to a degree, present in this theory.

3. Relevance in Aristotle's Syllogistic

A concept of logical relevance allegedly similar to that just presented has been spotted in various passages in the Aristotelian corpus by different authors. The suggestions include Aristotle's definition of the syllogism in I.1 of the *Prior Analytics* (in particular in the definition's requirement that the conclusion has to come about 'because of' (*dia*) the premises and 'from their being such' (*tōi tauta einai*) (Smith 2007)), and chapter I.23 (especially 40b30-41a20 (Smiley 1994, p. 30)), chapter I.25 (Woods 2001, p. 81; Woods and Irvine 2004, p. 65), as well as chapter II.17 (especially 65b9-12 and 65b21-32 (Smith 1989, pp. 210-11)) of the same work. It will

¹⁰ When we turn to Aristotle's investigation of the syllogism we will see that he too first argues for a condition equivalent to *variable-sharing* and then turns to the question whether or not a valid syllogism requires that all the terms and premises are used.

¹¹ For a similar set of conditions for relevance logic, cf. Mares and Meyer (2001, p. 286).

be the task of this section to evaluate these passages with an eye to determining whether the three conditions of relevance logic that we have set out above are present, i.e. whether the concept of relevance employed satisfies the *meaning content* and the *usage* conditions, as well as whether these conditions are imposed as restrictions on the validity of syllogisms.

The requirement that relevance is a matter of validity presents us with a number of difficulties, which we must first address. To begin with, discussing Aristotle's notion of validity is problematic for terminological reasons. Aristotle did not have a designated word or phrase to express the concept of validity; hence, we have to look for passages in which Aristotle describes the concept of logical consequence and from these passages construct the notion of validity that he employed.¹² Some commentators have suggested that the word *sullogismos* itself (in its more specific logical sense¹³) carries with it the notion of validity; every syllogism would then be valid and the phrase 'invalid syllogism' would be a contradiction in terms (Smith 2007). This view is to some degree plausible because Aristotle, in his investigation of the figures (e.g. I.4, 26a7-8), expresses what we would call invalidity by saying 'there will be no syllogism' (*ouk estai sullogismos*). Thus, the definition of syllogism that Aristotle provides at the beginning of the *Prior Analytics* suggests itself as a point of departure for the investigation of Aristotle's concept of validity. However, this definition is of a very general nature and will need to be supplemented and spelled out by subsequent passages.

¹² Of special interest in this context are passages in which Aristotle uses expressions such as 'ex *anagkēs sumbainei*', 'ei X, *anagkē* Y' or simply the word '*anagkaion*'.

¹³ *Sullogismos* can, of course, just mean 'argument' or 'reasoning' in an everyday sense (Liddell et al. 1996, *ad loc.*) besides its technical logical meaning, which we could translate as 'deduction'. I will use the word 'syllogism' in its technical meaning, i.e. as a technical term of logic as contrasted with an everyday use of the word 'argument' throughout the paper and disambiguate when necessary.

Secondly, certain elements of Aristotle's theory have given rise to the question whether the syllogistic stands in need of an underlying propositional logic (Łukasiewicz 1957, pp. 49; Rose 1968, pp. 53; Patzig 1968, ch. 5; Corcoran 1974b, pp. 92-93; Slater 1979), for example the so-called conversions Aristotle introduces in I.2, the reduction of imperfect syllogisms to perfect syllogisms that make use of these conversions and, thirdly, the hypothetical syllogism (cf. (Lear 1980, pp. 34-53)). As far as the present project is concerned, the possibility that Aristotle's syllogistic requires an underlying propositional logic could lead to the following objection: if the syllogistic requires an underlying propositional logic, and if relevance – in the syllogistic – is ensured through (structural) features of the syllogism, then it appears that there is an element of Aristotle's logic, namely the underlying propositional logic, for which it has not been shown that it observes relevance-conditions.

To be sure, those interpretations that affirm that the syllogistic requires an underlying propositional logic have been met by forceful arguments to the contrary (Corcoran 1974b; Smiley 1973). The really pressing question for the present paper, however, is not whether or not the syllogistic *requires* an underlying propositional logic, but whether or not Aristotle accepted arguments as valid that are not syllogisms. For, even if the syllogistic does not require an underlying non-syllogistic logic, the acceptance of non-syllogistic arguments has important consequences for the notion of validity we can ascribe to Aristotle. Two views have been advanced regarding this issue. The first view (I will refer to this as the A-view) says that Aristotle's notion of validity is different from his notion of syllogistic validity (henceforth s-validity).¹⁴ The former is an unanalysed primitive, the latter a proper subproperty of validity. S-validity, the proponents of the A-view agree, might be

¹⁴ This has been claimed by, for instance, Frede (1974) and Woods (2001).

constrained by conditions of relevance, but at the same time validity, its superproperty, might not. Therefore, showing that s-validity displays good approximations to the conditions upheld in modern relevance logic is not sufficient for the claim that it is a relevance logic, for, as we have seen in the discussion of Anderson and Belnap, modern relevance logic requires that relevance is a part of validity. According to the other view (henceforth the B-view), Aristotle regarded ‘the theory of syllogistic as the most fundamental sort of reasoning’ (Corcoran 1974b, p. 97). According to the B-view, validity is identical with s-validity and Aristotle’s theory of formal consequence, his logic, is identical with the syllogistic. This, evidently, would allow us to draw much more far-reaching conclusions about the relation between the syllogistic and modern relevance logic.

Although it is clear that the question whether the A- or the B-view should be given precedence is instrumental for the question this paper tries to answer, the considerations and arguments put forward in what follows remain, I think, valuable even if it should turn out that there is more convincing evidence for the A-view. First, if Aristotle really accepted a notion of validity different from that of s-validity, it is still an interesting and important question whether and how precisely relevance is ensured within the theory of the syllogism. Secondly, investigating the syllogistic from the viewpoint of relevance contributes to distinguishing between two different versions of the A-view; whichever of these two versions one embraces also has consequences for the question whether Aristotle’s logic is a relevance logic or not. Some authors merely claim that Aristotle accepted arguments different from syllogisms as formally valid without specifying in detail what this validity amounts to (e.g. Frede 1974). In this version of the A-view, it is still possible that Aristotle’s logic is a relevance logic: if the subproperty s-validity incorporates relevance, its

superproperty might also incorporate relevance (although the latter does not follow from the former).¹⁵ If, on the other hand, it turns out that s-validity does not incorporate relevance, then surely its superproperty will not incorporate relevance; this result could then be seen as support for the second version of the A-view whose proponents hold that the superproperty validity must be classical validity (e.g. Woods 2004).

Since, then, the question whether the syllogistic incorporates relevance is of interest independently of whether the A- or the B-view should be given precedence, I will first analyse the theory of the syllogism and in particular s-validity. I will then consider the arguments for and against the A- and the B-view, respectively. If it turns out that s-validity incorporates relevance, and if it turns out that there is stronger evidence for the B-view (i.e. that s-validity, for Aristotle, is validity), then we can proceed to draw the conclusion that Aristotle's logic is a relevance logic à la Anderson and Belnap. If, on the other hand, the evidence for the A-view is stronger, then we cannot conclude this, but only that s-validity incorporates relevance and that validity might incorporate it.

The first step in our search for relevance-properties in Aristotle's syllogistic is to take account of his definition or characterisation of the syllogism in *Prior Analytics*

I.1:

¹⁵ I will argue that the way in which syllogisms enforce relevance is peculiar to syllogistic form. The superproperty validity, applying to arguments different from syllogisms, cannot therefore incorporate relevance in the same way. This, however, does not prevent validity from being relevant, as one kind of logic might incorporate the concept of relevance in a way appropriate to that logic, while another kind of logic might have to take a different route to ensure that the same concept is part of its notion of validity. Consider, for instance, the case of a relevant predicate logic; cf. (Dunn 1987; Anderson et al. 1992, §74).

A syllogism is a discourse in which, certain things being stated (*tethentōn tinōn*), something other than what was supposed (*tōn keimenōn*) follows of necessity (*ex anagkēs sumbainei*) from their being so (*tōi tauta einai*). By ‘from their being so’, I mean ‘follow in virtue of them’ (*dia tauta sumbainein*), and by ‘follow in virtue of them’ I mean ‘needing no further term from outside’ (*mēdenos exothen horou prosdein*) in order for the necessity to come about (*to genesthai to anagkaion*). (24b19-22)

Apart from the idea already referred to, namely that the term *sullogismos* itself seems to carry the notion of (s-)validity within it, the text of the definition gives us a further reason to accept it as a statement about (s-)validity inasmuch as it clearly concerns consequence (*ex anagkēs sumbainei*), and, more precisely, logical consequence, as is clear by the definition’s mention of assumptions and statements (*tethentōn tinōn, tōn keimenōn*).

Smith (2007) suggested we understand the phrase ‘from their being so’ (*tōi tauta einai*) as an indication that the syllogistic incorporates a notion of relevance. Aristotle explains this part of the definition with two subsequent statements. The first explicates the dative of *tōi tauta* as a causal dative, ‘in virtue of’ (*dia*); the second explains ‘in virtue of them’ to mean that ‘no other terms from outside’ need to be introduced in order to ‘bring the necessity about’. Taken at face value, Aristotle seems to say no more than that if the premises feature the terms A, B and C, then the deduction of the conclusion may not rely on a term D, though it might rely on additional logical machinery. This, of course, is very general and will need to be explicated further. But we may note that from the text of this passage alone we cannot conclude that Aristotle demands that the terms of the premises *have to be used*, nor

that the conclusion and the premises have to *share terms*. Consider, for instance, the following argument:

Premise 1: All A is A

Premise 2: All B is B

Conclusion: All C is C

Would this argument be s-valid if we restrict ourselves to the definition quoted above as Aristotle's account of s-validity? It is a discourse and it satisfies the condition that the conclusion is 'something other than what has been assumed'.¹⁶ Whether the condition, which states that the conclusion follows necessarily, is satisfied, depends on how we understand *ex anagkēs sumbainei*. Within the classical framework, we would say that the conclusion does indeed follow necessarily; for, since it is impossible that the conclusion is false while all the premises are true, the argument is classically valid. Something similar could be said regarding the *dia tauta* condition. Interpreted from a classical viewpoint, the validity of the argument depends – at least partly – on the premises (is valid because of them, *dia tauta*); for, since the premises are true and the conclusion is true, the argument is valid.¹⁷ Hence, the text of the definition as it stands cannot decide the question whether the syllogistic is a relevance logic, because the answer to this question depends, precisely, on how we interpret the conditions of the definition. Aristotle's subsequent discussion of s-validity will have

¹⁶ It is, however, unclear whether the individual statements are well-formed. Aristotle's remarks on the form of premises up to this point of the *Prior Analytics* do not expressly forbid statements such as 'All A is A' (cf. 24a16-7 and footnote 19). In my view, Aristotle ultimately rejects such statements, but this question does not affect my point here. See (Corcoran 1974b, pp. 96 and 99).

¹⁷ In this particular example, since the conclusion is necessarily true, one could say that the validity does not depend on the particular premises used. But even if this is not the case, it becomes obvious that the validity depends (partly) on the premises, for an argument may never lead from truth to falsity. Hence, if we have an argument with a false conclusion, its validity depends on the falsity of its premises and therefore its validity could be said, from a classical viewpoint, to be *dia tauta* the premises.

to explicate the definition in order for us to be able to determine whether it incorporates a concept of relevance or not.

After the first chapter in which, in addition to the definition of the syllogism, Aristotle defines other fundamental terms like ‘proposition’ (*protasis*), ‘term’ (*horon*), ‘perfect’ (*teleion*) and ‘imperfect’ (*atelē*) syllogism, the *Prior Analytics* proceed thus: in the second and third chapter, Aristotle introduces conversions, the fourth chapter is dedicated to showing which moods of the first figure are s-valid and which are s-invalid, the fifth and sixth chapter do the same with the second and third figures, respectively. The seventh chapter begins to draw general conclusions of, as we would say, a metalogical character, which are continued in chapters 23 and 25, interrupted by the discussion of modal syllogisms in chapters 8-22. Chapters four to seven and 23 and 25 are therefore the most likely to contain important material for the present study.

Aristotle begins his investigation of the moods by assuming that a syllogism (i) relates three distinct terms that are distributed over two premises and one conclusion, such that (ii) a premise relates two distinct terms, (iii) a premise is not repeated, (iv) one term appears in two premises and (v) the conclusion relates the two non-repeating terms from the first and the second premise.¹⁸ If a syllogism is constructed from exactly three distinct terms, exactly two premises and one conclusion, exactly one middle term, and also satisfies the other conditions, I will call it a minimal syllogism, for reasons that will become apparent later. Likewise it will become clear later that there is another kind of valid syllogism that satisfies the five conditions, but contains more terms and premises than the minimal syllogism. The

¹⁸ Aristotle does not state these conditions explicitly. As a matter of fact, he investigates premise-pairs; but from his actual responses to a particular premise-pair we see that he also has certain conditions in mind relating to the conclusions (e.g. he does not consider syllogisms of the form: all A is B, all B is C, therefore all D is E.)

relation between minimal and non-minimal syllogisms is, as we will see, of crucial importance to the question of relevance.

In chapters 4-6, Aristotle systematically investigates all syllogisms that can be formed in compliance with the definition of a minimal syllogism by forming all possible premise-pairs according to his stipulations of the form of a proposition and he considers whether a conclusion necessarily follows.¹⁹ Each individual pair of premises receives one of three possible responses: either Aristotle states that the mood is evidently s-valid (for the perfect moods of the first figure), or he proceeds to prove the s-validity (for the imperfect moods) or he employs a countermodel-technique to show the mood's s-invalidity. Aristotle's proofs of the imperfect moods depend on the perfect moods and on the conversions and can either be direct or take the form of a *reductio* argument or involve *ekthesis*.²⁰

In chapter 7, Aristotle demonstrates that all valid syllogisms are shown to be valid by means of the first figure and indeed that it is possible to show that all valid syllogisms, even the two perfect particular ones (Darii and Ferio), can be led back or be reduced to (*anagein eis*) the two universal moods of the first figure, i.e. Barbara and Celarent. We note that the set of valid syllogisms Aristotle is investigating in I.4-6 (and indeed also those shown to be invalid), satisfy a condition equivalent to *variable-sharing* (i.e. Anderson and Belnap's necessary condition) *per definitionem*, if, as is plausible, meaning content in a term logic such as the syllogistic is captured by the terms (since Aristotle uses letters to stand for terms, I will call this condition

¹⁹ Aristotle, in I.1, 24a16-b12, stipulates that the propositions that serve either as premises or as conclusion have to be statements that affirm or deny something of something (*tinokata tinokata*) either universally (*katholou*) or particularly (*kata meros*). The phrase *tinokata tinokata* does not preclude self-predications, but since it is very likely that we can ignore this case, the different kinds of statements that are possible are: universal affirmative (All A is B), universal negative (No A is B), particular affirmative (Some A is B), and particular negative (Some A is not B).

²⁰ It has been argued that the proofs of the s-validity of the imperfect moods require a logic of propositions. For a rebuttal of this claim, see Corcoran (1974b); also see Patzig (1968, ch. 5).

*letter-sharing*²¹). For, one of the implicit defining conditions of the set is that the terms are distributed such that the terms forming the conclusion are also part of the premises. If it were possible to show that non-minimal valid syllogisms are valid because of valid minimal syllogisms, and if it were further possible to show that there are no other valid syllogisms apart from valid minimal syllogisms and those non-minimal syllogisms that are valid because of minimal syllogisms, then the syllogism as such could be said to satisfy the letter-sharing condition. Of course, this is merely a necessary condition and, hence, we do not have to be concerned that the syllogisms of the initial set shown to be invalid also satisfy this condition. Further, it will be necessary to show that the usage condition is satisfied as well. Here the question will arise whether Aristotle thinks that this condition, if it applies, is a necessary and sufficient condition, like in Anderson and Belnap, or just a necessary condition.

We will have to consider, therefore, whether the *Prior Analytics* contains arguments that show (i) that all non-minimal valid syllogisms are valid because of valid minimal syllogisms, and (ii) that there are no other valid syllogisms apart from valid minimal syllogisms and those non-minimal syllogisms that are valid because of valid minimal syllogisms. If (i) and (ii) can be established, it follows that the letter-sharing property that was satisfied by the minimal syllogism because of its structural requirements (number of terms and premises, distribution of terms) will also be true for those non-minimal syllogisms that are valid because of minimal syllogisms. We must also consider whether (iii) an argument can be found that establishes that valid minimal syllogisms satisfy the usage-condition, and if so whether this condition is understood as necessary only or as both necessary and sufficient.

²¹ There is a dispute over the question whether Aristotle's letters are variables or not. Łukasiewicz (1957, pp. 7-8), Corcoran (1974b, p. 100), and others believe that they are variables; Barnes (2012, p. 99, n. 139), Frede (1974, p. 19) and others believe that they are not or that it is at least questionable whether they are. Cf. also Kirwan (1978, pp. 3-8) and Barnes (2007, pp. 337-59).

Chapters I.23 and I.25 contain arguments that can be interpreted as establishing or at least beginning to establish all three points. In chapter 23, Aristotle argues that the structure of the minimal syllogism is a necessary condition for its s-validity by showing that a syllogism needs at least two distinct premises and at least three distinct terms; further, he argues that syllogisms are in need of a middle term, i.e. a term appearing in both premises. From these results he concludes that all syllogisms that can be generated with a view to these conditions are those considered in the initial set, i.e. the set he considers in I.4-6.

In chapter 25, Aristotle argues that there is no conclusion that could not be derived from exactly two premises (or exactly three terms), which are structured in the way he argued they must be in chapter 23, by showing that whenever a conclusion seems to require more than two premises (or three terms), we are either dealing with several syllogisms or that the conclusion can be deduced from just two of the premises that were supposed to be necessary to deduce the conclusion. The arguments of chapters 23 and 25 will allow us to draw some conclusions concerning the letter-sharing and usage conditions, but only for those syllogisms considered in the initial set, not for syllogisms with more premises and terms. It seems, however, that the purpose of Aristotle's arguments in I.23 and I.25 is also to show that valid finite arbitrarily long syllogisms can be analysed into valid minimal syllogisms and that invalid finite arbitrarily long syllogisms cannot be analysed into valid minimal syllogisms. If we understand this as a requirement (i.e. that a non-minimal syllogism is valid if and only if it can be analysed into valid minimal syllogisms), then our conclusions can be interpreted as affecting the notion of s-validity itself.

I will turn, then, to I.23. After stating that the conclusion of every syllogism must be a statement in which an attribute is said to belong (or not to belong) in the universal (or in the particular) sense to some subject, and that the proof of this must be either direct or indirect, Aristotle advances the following argument about direct proofs (40b30-41a20). If we suppose that the conclusion of an argument is AB,²² then it is necessary to assume something from which this conclusion is supposed to follow; for otherwise, we clearly have no argument at all, but just a statement. If we assume that AB follows from a premise that states AB, then, Aristotle says, ‘the initial thing will have been taken’ (40b32-3). The argument under consideration, thus, is $AB \vdash AB$. An argument of the form $\Pi \vdash \Pi$ is, of course, circular, and although circular arguments may be rejected for several reasons, they are generally considered to be logically valid.²³ However, if we take into consideration the definition of the syllogism, we have reason to believe that Aristotle is here saying that circular arguments are at least not s-valid, for the definition states that in a syllogism ‘something other than (*heteron ti*) what was supposed’ follows and this is evidently not the case in circular arguments.²⁴

The next case Aristotle considers is that AC is taken as a premise, but that nothing is assumed to apply to either A or C (i.e. there is no further premise AX or CX). The proposed argument is, then, $AC \vdash AB$. Again, Aristotle says that ‘there will

²² In what follows, I will use different notations depending on the purpose of the argument as well as to stay as close to Aristotle’s text as possible. In Aristotle, letters can stand either for propositions or for the terms the propositions relate to each other. In my notation, Latin uppercase letters always stand for terms and I will usually indicate the kind of relation that obtains between two such terms, designated by the Latin lowercase letters a, e, i, and o (see footnote 19). I will use lowercase x as a variable ranging over these letters, hence AxB is either AaB , AeB , AiB , or AoB . I may also omit the x and write AB when it is of no importance whether the statement is AxB or BxA . Greek uppercase letters, on the other hand, stand for the propositions that can occupy premise- or conclusion-position in an argument: $\Pi, \Psi \vdash \Theta$ can stand, e.g., for AxB , BxC , AxC .

²³ Circular arguments are classically valid; in the field of relevance logic, there are some systems in which circular arguments are also valid and some in which they are not. Cf. Mares (2012).

²⁴ At any rate, the idea that in a circular argument no *deduction* has taken place might be thought to be an intuitive judgment.

be no syllogism'. Explaining this passage, Alexander of Aphrodisias refers to the definition of the syllogism and in particular to the plural 'certain things being stated' which he takes as a demand for more than one premise and hence as grounds to rule out $AC \vdash AB$.²⁵ However, Aristotle's own explanation (*gar*, 40b35) is that in such a case 'nothing will follow of necessity'; thus, Aristotle might indeed be referring to the definition, but to a part different from the one Alexander suggests.²⁶ We would have, then, the following argument: since every syllogism necessitates a conclusion and since no conclusion is necessitated by the premise AC, an argument that only has AC as a premise is not a syllogism.

But why is it that in the case under consideration nothing follows of necessity? One way to make sense of Aristotle's remark is to understand it against the background of the countermodel-technique I mentioned before, for when Aristotle first employs countermodels, it is precisely with the aim of showing a lack of necessitation. The first mood Aristotle shows to be invalid in his analysis of the figures is characterized by the premise-set AaB, BeC. Assuming these premises, 'no conclusion follows necessarily from their being so' (26a4-5). In order to establish this claim, Aristotle proceeds to show that the premise-pair AaB, BeC allows for term-substitution such that the conclusion could be either an a- or an e-statement: the first would be the case if the three terms that are related in the syllogism are animal, man and horse; the latter would be the case if the three terms are animal, man and stone.

²⁵ Alexander, in *APr.*, 257.8-13. Smith (1989, p. 140) thinks that this might be the best explanation we can get.

²⁶ Of course, the two parts might not be independent of each other. In fact, it is precisely Aristotle's aim in this passage to show that if only one premise is assumed, there will be no syllogism. If we follow Alexander, we have to accept that syllogisms are multi-premised because of Aristotle's stipulation in the definition of the syllogism. Aristotle's own explanation, however, indicates that the demand that syllogisms have to be multi-premised is a consequence of the demand that syllogisms necessitate a conclusion. Also see section 4 below on how this passage relates to the question whether or not Aristotle accepted a validity different from s-validity.

This alone might be enough to show that no conclusion is necessitated if we have a premise-pair AaB , BeC , but it is possible to prove this more formally, as Patzig does. Patzig first gives necessary and sufficient conditions for a premise-pair (p) to belong to the class of pairs from which a conclusion can be s-validly deduced (P) : $p \in P \leftrightarrow [(A, B, C) (p \rightarrow AaC) \text{ or } (A, B, C) (p \rightarrow AeC) \text{ or } (A, B, C) (p \rightarrow AiC) \text{ or } (A, B, C) (p \rightarrow AoC)]$. On the other hand, pairs that belong to the class from which no conclusion can be s-validly deduced (I) are characterized by the negation of this expression; hence, membership to this class is defined thus: $p \in I \leftrightarrow [(\exists A, B, C) (p \& AoC) \text{ and } (\exists A, B, C) (p \& AiC) \text{ and } (\exists A, B, C) (p \& AeC) \text{ and } (\exists A, B, C) (p \& AaC)]$. Patzig then shows that in order to prove that p is a member of I all that has to be done is to give a triad that makes p and AaC true and a different triad that makes p and AeC true, which is exactly what Aristotle does. For, if AaC is true, AiC is also true, and if AeC is true, AoC is true (Patzig 1968, pp. 175-176). Returning to Aristotle's analysis in I.23, we can now apply a similar method to the argument under consideration, $AC \vdash AB$. AC is the sole member of our premise-set p and AB is the conclusion. It is easy to see that we can find term-triads that make p and AaB as well as p and AeB true, and hence also p and AiB and p and AoB . Hence, $AC \vdash AB$ lacks the required necessity and Aristotle's intermediary result is, therefore, that we need to assume at least one more premise.

The case he considers next is the following: we assume another premise in addition to AC and this premise is either AX or CX ; in this case we can indeed get a conclusion (namely, either CX or AX), but it will not be the conclusion we sought, AB . Aristotle again simply states that no conclusion AB can be deduced in this case, and again we could refer to the explanation based on a lack of necessitation, given above. The consequence of the consideration of these cases is that we need a premise

that contains the term B; if we continue to think of the first premise of our syllogism as AC, and AB as the desired conclusion, then the second premise has to be CB, so that we get to AC, CB \vdash AB. This is a major result of Aristotle's argument and he proceeds to formulate²⁷ it as a general (*holos*) theorem: 'we will never have a syllogism proving that one thing applies to another if we do not assume a middle, which is related in some way by predication to each of the other two.' (41a2-4) Thus, Aristotle here states that a middle term is necessary.

Aristotle draws the following conclusion from the whole argument: since a middle term is required in any valid syllogism, and since there are only three ways of forming a syllogism with two premises, one conclusion and three distinct terms, such that it fulfils the requirements the argument has revealed, it follows that every (*panta*) valid syllogism is brought about by one of the three figures (41a13-8). For, the three possibilities to form a syllogism with two premises and three distinct terms with one term appearing in both premises are: (i) predicating A of B and B of C (i.e. the first figure) or (ii) A of B and A of C (the second figure) or (iii) A of C and B of C (the third figure). Hence, what before was true only *per definitionem*, namely that the syllogisms considered in I.4-6 all satisfied the letter-sharing condition, is now supported by an argument: there are no valid syllogisms that do not at least consist of two premises and one conclusion with three distinct terms; one of these terms appears in both premises, while the conclusion consists of the two remaining terms; hence, *letter-sharing* is indeed a necessary condition of the validity of minimal syllogisms.

Indeed, Aristotle's argument allows us to make a more general claim: since it has

²⁷ Ross reads *eipomen*, which would imply that Aristotle has already stated such a theorem (Ross (1949, p. 371) thinks it is implicitly stated in I.4-6). But since this is really the conclusion of the argument Aristotle has just presented, reading *eipōmen* with the first hand of ms. A and ms. B, as Ebert and Nortmann (2007, p. 742 n. 1) suggest, seems to make more sense. Aristotle's explanation (*gar*) immediately following the theorem seems to do no more than summarize the steps of the argument he has just given.

been shown that no conclusion can be deduced at all if a syllogism does not satisfy certain requirements concerning the number and distribution of terms, even a syllogism with more terms and premises than the minimal syllogism will satisfy the letter-sharing condition. For, the letter-sharing condition is satisfied if at least one letter is shared by both premise-set and conclusion.

In my view, this result also allows us to draw a preliminary conclusion concerning the usage condition. Aristotle has stated that one premise is never enough to derive a conclusion and I have suggested that we should understand this claim against the background of his countermodel-technique. Now, if one premise is not enough to derive a conclusion, it stands to reason that in a two-premise syllogism both premises have to be actually used. When we turn to chapter II.17 we will see candidates of syllogisms of this form: $AB, BC, CD \vdash BD$, a syllogism in which arguably the premise AB is not used and which could be restated as $BC, CD \vdash BD$. Now, if a similar syllogism could be formed with just two premises (i.e. a two-premise syllogism in which only one premise is actually used), we could restate the syllogism as having only one premise, but Aristotle has just concluded that this is not possible. Hence, it appears that both premises in a minimal syllogism contribute crucially to the deduction of the conclusion and that, therefore, the argument in I.23 also seems to contribute to a proof that *usage* is at least a necessary condition for the validity of syllogisms.²⁸

But at this point *usage* is not completely established as a necessary condition. This would only be the case if Aristotle held that the only valid syllogisms are those with the structural requirements of minimal syllogisms. But since it is beyond doubt

²⁸ In the argument I have presented, Aristotle only considers direct arguments. He then (41a21-b5) adds an argument to show that the same applies to *reductio* arguments, which I will not present here.

that he accepts arguments with more premises than two, we cannot yet decide whether *usage* is a necessary condition of s-validity. However, if Aristotle were to show that finite arbitrarily long syllogisms are valid if and only if they can be analysed into minimal syllogisms, it seems that we could draw such a conclusion.

I believe that I.23 and I.25 contain arguments that can be understood as entailing such a reductive claim. Chapter 23 establishes that the structural features of minimal syllogisms are necessary for a deduction to come about; chapter 25 argues that there are no conclusions that require more premises or terms than the minimal syllogism requires. The purpose of these chapters cannot be to establish the validity of the syllogisms accepted as valid in I.4-6, for, in Aristotle's view, this part of the project has been completed. What is missing, however, is an answer to the question: which of those arguments are s-valid that have more premises and terms than minimal syllogisms? If Aristotle succeeds in showing that there are no conclusions requiring more premises (or terms) than minimal syllogisms, and since he has already shown that syllogisms at least have to satisfy the structural features minimal syllogisms satisfy, he can then argue that finite arbitrarily long syllogisms can be analysed into syllogisms of the minimal type; for, if the latter were not the case, a contradiction with the results of either I.23 or I.25 would arise. If, moreover, analysability into minimal syllogisms is understood as a requirement for the validity of non-minimal syllogisms, then we are allowed to extend the claim concerning the status of the *usage* condition from syllogisms of the minimal type to syllogisms in general.

Aristotle has shown in I.23 that none of the terms of the premises are dispensable for a given conclusion to come about, but it is still open to question whether a syllogism requires more than three terms or more than two premises for its

conclusion to come about. Chapter 25 takes up this question and announces at the beginning that ‘it is clear also that every (*pasa*) demonstration will come about through three terms and no more’ (41b36-7).²⁹ Aristotle first distinguishes two examples, in which a deduction seems to require more than three terms: (i) $\Pi, \Psi \vdash \Theta$ and $\Gamma, \Delta \vdash \Theta$; (ii) $\Delta, \Theta \vdash \Pi$ and $\Phi, \Sigma \vdash \Psi$ and $\Pi, \Psi \vdash \Theta$.³⁰ In case (i) the conclusion Θ comes about both through Π and Ψ as well as through Γ and Δ ; but this, Aristotle explains, is not one syllogism, but several and, hence, his thesis that a deduction requires no more than three terms is not affected. The same is true for case (ii), in which the premises for the proof $\Pi, \Psi \vdash \Theta$ are themselves proven by two further syllogisms with the premises Δ and Θ or Φ and Σ , respectively. Obviously, we again have several syllogisms and not one. Then, Aristotle goes on to consider an assumed syllogism of the form $\Pi, \Psi, \Gamma, \Delta \vdash \Theta$. He will systematically investigate this syllogism by differentiating several cases and answering each of them either by (a) asserting that we deal with a case like (i) above, i.e. that the same conclusion can come about through different middle terms and that, hence, we deal with several syllogisms; or (b) by asserting that we deal with a case like (ii) above, i.e. the premises are themselves proven from other premises and that again we are dealing with several syllogisms; or (c) by asserting that a proposition will be concluded that is neither $\Pi, \Psi, \Gamma, \Delta$ or Θ and that, hence, we have several syllogisms that are

²⁹ Aristotle’s formulation of the theorem to be established in this chapter demands a short explanation. Given the undeniable fact that Aristotle allows for syllogisms with more than two premises, we should not understand the theorem to say that there will be no syllogisms with more than two premises, but rather that there are no deductions (in the sense of the conclusion, i.e. the result, of a syllogism) which require more than two premises: any given conclusion, the theorem says, can be stated as the conclusion of a syllogisms with two premises and no more. The occurrence of the word *apodeixis* (demonstration) instead of *sullogismos* is curious but should not give rise to any concerns about the theorem: Aristotle will talk of *sullogismoi* in the remainder of the chapter and in any case he has stated before that every demonstration is a syllogism (but not *vice versa*), so the result of his discussion will apply to syllogisms.

³⁰ In I.25 the letters stand for syllogistic statements (i.e. propositions), so either for a premise or for a conclusion. For the notation, see footnote 22.

unconnected; or (d) by asserting that a pair of premises will not yield any conclusion and, hence, that we either have no syllogism at all, or one syllogism and some remaining statements that are not related to it. For the sake of brevity, and since it is generally accepted that this part of Aristotle's proof is successful in showing that if Θ is a conclusion, then the syllogism, which has Θ as a conclusion requires only three terms and two premises, I will not reproduce the details.³¹

The question that is decisive for our purposes, however, is whether the argument $\Pi, \Psi, \Gamma, \Delta \vdash \Theta$ is s-valid if, in fact, Θ follows from Π and Ψ alone. It has been remarked in the scholarly literature on this chapter that here Aristotle comes close to explicitly stating a condition similar or identical to *usage* (Woods 2001; Woods and Irvine 2004). First, Aristotle twice (42a23 and 29) says that premises like Γ and Δ in a syllogism $\Pi, \Psi, \Gamma, \Delta \vdash \Theta$, where, in fact, Θ follows from Π and Ψ alone, are 'in vain' (*matēn*),³² moreover, at 42a39 he says that in these cases 'more than necessary for the [deduction of the, PS] thesis has been taken (*pleiō tōn anagkaiōn*)'.

Are these statements, on their own, sufficient for deciding the question whether *usage* is a condition of s-validity? This does not seem to be the case. For, both advocates of classical validity and those of non-classical validity can provide interpretations of these statements that match their respective views. From a classical point of view, a conclusion is validly deduced even if only some of the premises are required and others are superfluous: classical validity is monotonic. Advocates of non-classical validity, on the other hand, understand the statements as indicating precisely the opposite: none of the premises can be superfluous and validity is non-monotonic. Thus, these statements in isolation cannot decide the question whether

³¹ For an extensive analysis, see Ebert and Nortmann (2007, p. 756-759).

³² Cf. also I.32, 47a16-9, where Aristotle, in addition to calling premises *matēn* uses the phrase *ti periergon* to describe them as 'superfluous' assumptions.

usage is a condition of s-validity. But if we understand I.23 and I.25 as (part of) an argument showing that every finite arbitrarily long syllogism is valid if and only if it can be analysed into minimal syllogisms (i.e. that analysability into minimal syllogisms is a necessary condition for the validity of syllogisms with more terms and premises than minimal syllogisms), then we can conclude that *usage* is a condition of s-validity.

I will finally turn to chapter II.17, which is concerned with a fallacy sometimes called ‘the false-cause fallacy’. Aristotle’s treatment of this fallacy is the closest he ever comes to talking explicitly about unused premises; and although the chapter is, for this reason, of central importance to the project of this paper, it is difficult to establish an unambiguous interpretation for or against the thesis that *usage* is a necessary condition of s-validity. Aristotle distinguishes two principal forms in which the fallacy can occur, an obvious and a less obvious one. In the obvious form (65b13-21) the individual statements of an argument have no connection (*sunechēs*) whatsoever. In the example Aristotle provides, someone aims to show that the diagonal of the square is incommensurable with the sides. In order to do so, he first assumes that it is commensurable; he then somehow imports Zeno’s argument that motion is impossible; having thus arrived at an impossible conclusion, he claims that the original assumption (that the diagonal of the square is commensurable) is false and that, hence, its contradictory is true. In the less obvious form (65b21-32) there are connections between the individual statements, but the conclusion is not connected with the original assumption.

Before looking at the second form in more detail (Aristotle does not spend any time discussing the first form, and neither will I), it is worth taking note of Aristotle’s introductory remarks about the fallacy and its non-significance in direct syllogisms. If

something is refuted directly (i.e., I take it, AaB, for instance, is shown to be false by a direct syllogism concluding to AoB) by means of the terms A, B, C, it cannot be maintained that the syllogism does not depend on the assumption,

because we say that ‘not because of *this*’ arises when, if *this* is removed, the syllogism is brought to a conclusion just as much (*mēden hēttōn perainētai*), which is not the case in direct syllogisms (*en tois deiktikois*); for if the assumption is removed, the syllogism related to it (*pros tautēn*) will not come about. Thus it is clear that we say ‘not because of *this*’ in *reductio ad impossibile* arguments when the original assumption is so related to the impossible conclusion that the latter follows just as much (*ouden hēttōn sumbainein*) both in the case that the assumption is present and in the case that it is not present. (65b6-12)

There are two interesting statements in this passage. First, Aristotle seems to say that there are never unused premises in direct syllogisms: contrary to false-cause infected *reductio* arguments in which the syllogism comes about after the original assumption has been removed, direct syllogisms will not come about if the assumption is removed. This sounds like Aristotle is indeed saying that a syllogism of the form $\Pi, \Psi, \Gamma \vdash \Delta$ is invalid in case Δ follows from Ψ and Γ alone, for clearly, even if we take away Π , the conclusion Δ will, in this case, come about. On the other hand, it would be surprising if Aristotle were to make such a distinction between direct and indirect syllogisms: if he claimed here that a direct syllogism is valid only if the removal of any of its premises leads to its invalidity, why would this not likewise and immediately apply to indirect syllogisms? Moreover, Aristotle’s explanation why the removal of an assumption in a direct syllogism is not possible without the syllogism

ceasing to reach its conclusion, is curiously phrased as ‘the syllogism relating to the assumption (*pros tautēn*) [i.e. the one which was removed, PS] will not come about’ (65b8-9). Obviously this is true: if an assumption is removed, the conclusion of the syllogism cannot relate to the assumption; but what is perplexing is that, as we will see, the same is true for indirect syllogisms and, hence, it does not seem to be a good criterion by which to distinguish between direct and indirect syllogisms. I think that, in the face of the problems and uncertainties of this passage, we are not entitled to count it as evidence for or against *usage* as a condition of s-validity.

The second remarkable feature of this passage is Aristotle’s usage of the phrases ‘*ouden hētton*’ or ‘*mēden hētton*’, which I translated ‘just as much’. Aristotle says five times that a certain conclusion follows (*sumbainein*) ‘just as much’ whether a certain assumption is present or not (65b7, 65b12, 65b27, 65b31 and 66a8). If this is interpreted as saying that the syllogism $\Pi, \Psi, \Gamma \vdash \Delta$ is valid even if Δ in fact follows from Ψ and Γ alone, then this passage could be counted as evidence for Aristotle’s acceptance of monotonicity. Concluding that the syllogistic is monotonic would, of course, decide the question whether or not the syllogistic is a relevance logic, for monotonicity is incompatible with *usage* and hence with a necessary condition of relevance logic. If the ‘*ouden hētton*’ or ‘*mēden hētton*’ is not an indication of monotonicity, how else can we understand it? I think it is not impossible that all Aristotle is trying to show by using this phrase is that a given conclusion requires fewer premises than actually stated, i.e. the conclusion does not need a certain premise or a number of premises. It would not be unnatural to point out to someone that his argument does not rely on a certain premise by saying ‘your conclusion will also (or just as much) come about if you remove that premise’ without thereby subscribing to monotonicity.

I will now turn to the non-obvious kind of fallacy (65b21-32). The sort of syllogism Aristotle has in mind for this form is $AB, BC, CD \vdash BD$, where BD is impossible or absurd and it is hence concluded that AB is false; the problem with this syllogism is that BD follows from BC and CD and that therefore, although we have a connection to the first premise via the term B , we should not be able to conclude that AB is false, but, assuming that BD is indeed impossible and CD is true, that it is BC which has been shown to be false. Aristotle's response to this problem is the following:

But the impossibility must (*dei*) be connected to the original terms, for in this way it will be because of (*dia*) the assumption, e.g. in the downwards direction, taking something connected to the term which is predicated; for if it is impossible that A belongs to D , then when A is removed, the falsity will no longer exist. (65b32-6)

First, we should note that in his description of $AB, BC, CD \vdash BD$ Aristotle twice says that the conclusion does not follow because of the assumption (*mē mentoi di' ekeinēn sumbainoi*, 65b22 and *ouk an eiē dia tēn ex archēs hupothesein*, 65b27) while here, considering the, in Aristotle's opinion, properly connected syllogism $AB, BC, CD \vdash AD$ he says that the conclusion does come about because of (*dia*) the assumption. $AB, BC, CD \vdash AD$, therefore, is said to comply with the definition of the syllogism (in particular, with its *dia*-condition), while $AB, BC, CD \vdash BD$ is said not to comply with it. Moreover, Aristotle explains why, in the latter case but not in the former, the conclusion comes about because of the assumption: if the conclusion were to come about because of the assumption, then a removal of the assumption should have an

effect on the coming about of the conclusion; or, in other words, if the conclusion depends on a certain set of assumptions, then a change in this set should have an effect on the conclusion, but if it does not depend on it, no effect is to be expected. The phrasing and the explanation might be seen as a reference to the definition of the syllogism, which demanded that the conclusion ‘follows because of (*dia sumbainein*)’ the premises; if this is indeed the case, we would have reason to reject the s-validity of $AB, BC, CD \vdash BD$ while accepting that of $AB, BC, CD \vdash AD$.

The results of Aristotle’s arguments in chapters I.23 and I.25, would, if correct, allow us to give a more formal explanation of the invalidity of the former and the validity of the latter syllogism. For, if we accept that analysability into minimal syllogisms is a necessary condition for the validity of non-minimal syllogisms, we have a criterion by which to judge the two syllogisms in question: for, while the latter syllogism is analysable, the former is not. The analysis, in order to be a proper analysis, has to satisfy the following two conditions: first, that the non-minimal syllogism and the minimal syllogisms terminate in the same conclusion; and secondly, that each premise of the non-minimal syllogism will appear in at least one minimal syllogism (consequently, all terms appearing in the non-minimal syllogism will also appear in the minimal syllogisms). The second part of the criterion (i.e. that the premises and, as a consequence, the terms of the non-minimal syllogism have to reappear in the minimal syllogisms) seems to be a reasonable demand if the process is to be a real analysis. For if it were the case that the minimal syllogisms would lack a premise that does appear in the non-minimal syllogism, then a synthesis of the minimal syllogisms into the non-minimal syllogism would require the addition of a premise from, as it were, outside. Likewise, the minimal syllogisms cannot be understood as an analysis of the non-minimal syllogism if the non-minimal syllogism

contains an element that does not appear in the minimal syllogisms or is not derivable from the minimal syllogisms.

However, this might give rise to the objection that the argument is circular. For, one might think that the second part of the criterion demands that the premises and terms are *used*; hence, it would not be the structural features of the minimal syllogism that enforce usage (and through which the non-minimal syllogism also satisfies the *usage*-condition); rather it would be the second part of the criterion of the process of transformation that enforces *usage*. This objection can be met by pointing to the difference between usage and appearance. The second part of the criterion of analysis does not demand that the premises are used, but merely that they reappear. This is not just a change of words to avoid the looming circularity. As we have seen in the discussion of modern relevance logic and its proponents' complaints about some classically valid deductions, the mere appearance of a premise in itself says nothing about its being used or not. Therefore, the demand that a premise has to reappear in the process of an analysis also goes no way towards establishing that this premise is actually used in deriving the conclusion.

Returning to the two syllogisms under consideration, $AB, BC, CD \vdash BD$ and $AB, BC, CD \vdash AD$, let us suppose that each individual pair of terms is connected with an a-predicate, so that we get the following two syllogisms: (i) $AaB, BaC, CaD \vdash BaD$ and (ii) $AaB, BaC, CaD \vdash AaD$, both of which, we note, have the same premises, but different conclusions. If analysability into minimal syllogisms is a condition of s-validity, and since Aristotle seems to reject (i) while he accepts (ii), we should expect the analysis of (i) into minimal syllogisms to fail at least one of the two parts of the criterion for successful analyses outlined above. Syllogism (ii), on the other hand, should satisfy the criterion, and indeed it does. For if we analyse (ii) into

the minimal syllogisms $AaB, BaC \vdash AaC$; $AaC, CaD \vdash AaD$, it is clear that, first, the final conclusion of these two minimal syllogisms is identical with the conclusion of the non-minimal syllogism and, secondly, that each and every premise of the non-minimal syllogism reappears in the minimal syllogisms. A transformation of (i) into minimals would, on the other hand, yield: $AaB, BaC \vdash AaC$; $AaC, CaD \vdash AaD$. This time, the criterion for a proper analysis is not satisfied because the final conclusion of the minimal syllogisms (AaD) is not identical with the conclusion of the non-minimal syllogism (BaD). Therefore, if analysability is a condition of s-validity, we can reject (i) but keep (ii).³³

It therefore seems that Aristotle recognized the danger unused premises can pose in the context of indirect arguments and that his previous analysis provides him with the formal means to fend off this danger and declare syllogisms with unused premises invalid. At the same time, he does not explicitly resort to the analysis of chapter I.23 and I.25 in order to do so, but it does not seem implausible that his remark that some conclusions come about ‘because of’ the assumptions while others only seem to do so, is supposed to lead in that direction; for we have seen that this element, originally introduced in the definition of the syllogism, played a crucial role in the analysis of I.23 and I.25 as well, where the ‘because-of’-condition is spelled out.

³³ Analysability is an answer (or part of an answer) to the question: what makes a complex argument valid? In natural deduction systems, in order to show that a complex argument is valid, it is necessary to show that each step of the argument is sanctioned by rules of which we know that they preserve validity. Analysis allows us to reduce a complex argument into simpler parts of which we know that they preserve validity. It is minimal syllogisms for which Aristotle shows whether or not they are s-valid and it is the structure of the minimal syllogism for which he presents an argument concerning the conditions of its s-validity. Clearly, he needs some formal way of linking the simple parts (the s-validity of which he presents an argument for) with more complex arguments, the possible s-validity of which he accepts. It should be noted that analysability is not an alternative to a natural deduction system (or, for that matter, to an axiomatic system), but rather a different way to express the same underlying idea.

4. S-validity and validity

We can summarize our evaluation up to this point as follows. It seems that Aristotle lays down a general description of s-validity in his definition of the syllogism. With this definition in the background he first investigates a set of syllogisms structured in the way I have called ‘minimal’. He establishes which syllogisms in this set are valid and establishes dependence relations between them (i.e. the dependence of the second and third figure on the first, and in particular on the two universal moods of the first figure). Aristotle then tries to show that the initial set of syllogisms was no arbitrary choice, but that the structural characteristics of the syllogisms of this set – three distinct terms distributed over two premises such that one term appears in both premises – are both the minimal requirements for a conclusion to come about (i.e. no conclusion can come about if a syllogism has fewer than two premises or fewer than three distinct terms or no middle term) as well the maximally necessary requirements for any given conclusion to come about (i.e. there is no conclusion that requires more than two premises or three distinct terms). This proof entails that any valid syllogism will satisfy *letter-sharing*; also, at least inasmuch as minimal syllogisms are concerned, *usage* has likewise been shown to be a necessary condition. Aristotle’s subsequent arguments, moreover, suggest that Aristotle makes the validity of non-minimal syllogisms dependent on minimal syllogisms (and eventually, of course, on the perfect syllogisms), which allowed us to extend the scope of the necessity of *usage* from minimal to non-minimal syllogisms. Finally, Aristotle’s treatment of the false-cause fallacy turned out less clear than desirable in either direction. Aristotle directly discusses the crucial case of unused premises, but does not explicitly refer to the formal solution that would have been available to him if my interpretation is correct. His mention of the crucial ‘because of the assumptions’-element of the

definition of the syllogism, however, might be seen as an indication in that direction. For, it is this element that, at least in part, drives his argument that leads to the thesis that the validity of non-minimal syllogisms depends on that of minimal syllogisms. It appears, therefore, that Aristotle's syllogistic satisfies the letter-sharing condition as well as the usage condition.

Contrary to modern relevance logic, *usage* is not a sufficient condition for s-validity, for the syllogisms shown to be invalid in I.4-6 also belong to the set whose defining characteristics allowed us to show that *usage* is a necessary condition. However, although *usage* is not a sufficient condition for s-validity, it is a sufficient condition for *letter-sharing*: any syllogisms that satisfy *usage* will also satisfy *letter-sharing*. *Letter-sharing*, on the other hand, does not entail *usage*: in syllogisms such as $AB, BC, CD \vdash BD$ the set of premises and the conclusion share letters, but they do not satisfy the usage condition.

We have reason to believe, therefore, that the syllogistic displays good approximations to the constraints imposed in modern relevance logic. This in itself is an interesting result; but in order to proceed to the claim that the syllogistic is a relevance logic à la Anderson and Belnap, it is necessary to consider the evidence for and against the thesis that s-validity is, for Aristotle, merely a subproperty of validity, or, on the other hand, identical with validity. For, as we have seen in the presentation of modern relevance logic, Anderson and Belnap urge that relevance is a condition of validity itself. If Aristotle accepted non-syllogistic arguments as valid, the possibility arises that relevance is not a condition of validity itself, but only of s-validity, since up to now we have only presented an argument that shows that syllogisms impose relevance-conditions. At the beginning of section 3, I introduced two different views on this issue: the A-view states that Aristotle did accept arguments different from

sylogisms as formally valid. One version of the A-view refrains from spelling out what this notion of validity amounted to and is content to take validity as an unanalysed primitive, while a second version states that Aristotle's validity must be classical validity. The B-view, on the other hand, claims that (only) s-validity is validity for Aristotle.

I submit that I take the B-view to be the more natural view, and that if we are unable to find pressing evidence in favour of the A-view, we should maintain the B-view. The reason for this is twofold: first, Aristotle shows great interest in s-validity and he carefully investigates this notion. If he accepted a more general validity, it is highly perplexing that he undertook no attempts to investigate it as well. Secondly, it appears that ancient and medieval authors did not even consider that Aristotle accepted a validity different from s-validity. This can be seen, for instance, in these authors' presentations of the order of the *Organon*: the *Categories* are understood as investigating words, the *De Interpretatione* as investigating propositions, the *Prior Analytics* as investigating syllogisms and the *Posterior Analytics* as investigating demonstrations. The ancient authors understand the *Organon* to be complete, i.e. there is no gap between the *De Interpretatione* and the *Prior Analytics*, which would be the place where a more general notion of logical consequence would be investigated in this order.³⁴ Even 19th century commentators like Waitz (1844) do not appear to assent to the A-view. Only in the post Frege-Russel-Whitehead period, i.e. in the period characterised by the acceptance of classical validity as the Official view, do we find scholars who clearly and unequivocally embrace the A-view. This should make

³⁴ For presentations of the order of the *Organon* see, for instance, Simplicius, in *Cat.*, 14,33-15.8 or David the Invincible, *Commentary on Aristotle's Prior Analytics*, II,2-3. Due to the nature of the transmission of Stoic views, it is difficult to determine the latter's position regarding our present question, but it has been observed (Sorabji 2005, p. 250) that the Stoics and the Aristotelians rejected each other's examples of valid arguments. This, however, is not specific enough for our purposes.

us alert to the possibility that the A-view is anachronistic. To be sure, none of the points just mentioned can count as conclusive evidence in favour of the B-view. But, I think, it shifts the burden of proof to the advocates of the A-view.

The most general kind of argument that could be made in favour of the A-view is derived from various examples of arguments Aristotle makes and which he apparently regarded as valid, although they are not syllogisms, for instance because they have only one premise. This kind of evidence is, in my view, generally inconclusive, for just as it is common practice nowadays not to formalize each and every argument in order to make obvious any required derivations and to explicitly state all required premises, Aristotle also might have followed this practice.³⁵ We have to take into account the possibility that Aristotle believed that these arguments could be shown to be formally valid by turning them into proper syllogisms – even if this is factually wrong. For instance, it has been noted many times that the theory of the syllogism cannot account for the mathematics of even Aristotle's time (Mueller 1974, p. 37); yet Aristotle seems to be convinced of the contrary.³⁶ Some non-syllogistic arguments that Aristotle appears to accept as valid might allow for a transformation into syllogistic form and Aristotle might have accepted them as valid for precisely this reason. Other arguments do not allow for such a transformation, but Aristotle might still have believed that they can be transformed. Thus, it will not be possible to make a conclusive argument in favour of the A-view by appeal to examples of non-syllogistic arguments that can be found in the Aristotelian corpus.

Secondly, Frede has suggested that the definition of the syllogism itself can be seen to contain a distinction between validity and s-validity. According to Frede, the

³⁵ We will see below how Alexander employs this thought in his commentary on *Prior Analytics* I.32, an important chapter for the issue under consideration.

³⁶ Cf. *APo* I.14, 79a17-32. See Mendell (1998).

definition comprises two parts: the first part – ‘*sullogismos de esti logos en hōi tethentōn tinōn heteron ti tōn keimenōn ex anagkēs*’ (24b18-20) – defines ‘a class, or the class, of valid inferences’, while the second part – ‘(*sumbainei*) tōi tauta einai’ (24b19-20) – ‘serves to single out those valid inferences which are syllogisms’.

(Frede 1974, p. 20) This interpretation is untenable. Anything but reading the definition as a definition of a single thing, *sullogismos* (irrespective whether the word is understood to signify ‘syllogism’ or ‘valid argument’), and the conditions as conditions of a single thing, violates the clear meaning of the Greek and requires an elliptical reading that already assumes that the A-view is correct.

Thirdly, the strongest direct textual evidence for the A-view can probably be found in *Prior Analytics* I.32. In this chapter, Aristotle appears to be making a distinction between ‘following syllogistically’ and ‘following necessarily’ – a distinction that appears to match that of s-validity and validity. The two sentences from which such a view could be construed are almost identical in phrasing. In the first Aristotle says that ‘some [arguments] seem to syllogize (*dokousi sullogizesthai*) because of the necessary following (*dia to anagkaion ti sumbainein*) from the assumptions’ (47a23-4). Since some arguments merely seem to be syllogisms, we can confuse them with proper syllogisms, and we do so, as Aristotle explains in the second passage, ‘because of the necessary following of something (*dia to anagkaion ti sumbainein*) from the assumptions, because syllogisms are also necessary’ (47a32-3). Moreover, he says that ‘necessity (*to anagkaion*) is wider than syllogism, for every syllogism is necessary, but not every necessity is a syllogism’ (47a33-5). Since Aristotle seems to be saying, therefore, that there is a kind of logical consequence although this consequence is not syllogistic, it appears that Aristotle recognised a validity different from s-validity.

The evidence for the A-view that can be gathered from a reading of this chapter indeed appears strong. But I do not think it conclusive. First, the fact alone that Aristotle thinks that necessity has a wider scope than syllogistic necessity comes as no surprise if we consult *Metaphysics* Δ, where Aristotle distinguishes several kinds of necessity, some of which have nothing to do with logic.³⁷ Further, the *Posterior Analytics*, and especially chapter I.4 of that work, give the impression that Aristotle recognised a kind of necessity that is not syllogistic. Neither, however, does this necessity come from a different kind of logical derivation. It is not the necessity of a logical consequence Aristotle is talking about when he discusses the nature of the principles of demonstrations, but rather the necessity of a proposition that flows directly from an essential relation that is expressed in that proposition.³⁸ The confusion Aristotle is pointing to, then, could arise because of the fact that the conclusion is a necessary proposition although the necessity of that proposition is not that of logical consequence. Although this interpretation is, I think, possible, it requires us to read the phrase ‘*sumbainein ek tōn keimenōn*’ in an artificial way that differs from other occurrences of this phrase in the *Prior Analytics*. In this reading the phrase would not mean, as it does elsewhere, ‘follow from the assumptions’, but ‘follow after/upon the assumptions’, i.e. the conclusion simply appears sequentially after the assumptions, although it does not follow from them. This latter meaning of the Greek preposition *ek* is normal, yet we do not find it elsewhere in the logical works.

³⁷ *Met.* Δ V, 1015a20-b15.

³⁸ Cf. *APo* I.4, 73a21-b5. The fact that the necessity of the principles of demonstrations is not the necessity of logical consequence has nothing to do with the difference between syllogism and demonstration. It is, of course, possible to set up a syllogism that has as its conclusion a proposition that could serve as a principle in a demonstration. But the logical necessity this proposition receives from the fact that it is a conclusion of a valid argument is not the necessity that makes the proposition suitable for a demonstration. The kind of necessity that allows us to use a proposition as a principle in a demonstration cannot be, as Aristotle points out several times in the *Posterior Analytics*, that of a logical consequence.

A different and, in my opinion, more plausible explanation of Aristotle's meaning in I.32 can be found in the commentary of Alexander of Aphrodisias. Alexander shows an awareness of the issue under consideration when he notes that some readers might ask themselves how Aristotle's seeming acknowledgement of arguments in which something follows necessarily from the assumptions although these arguments are not syllogisms, relates to the definition of the syllogism (350,11-8.). His answer is based on the 'because-of' condition of the definition of the syllogism. Contrary to Frede's interpretation, however, Alexander does not arrive at the conclusion that Aristotle distinguished between validity and s-validity; rather, the non-syllogistic arguments Aristotle refers to have, in Alexander's reading, the status of enthymemes.

Alexander's interpretation takes its cue from Aristotle's introduction of the chapter, in which Aristotle announces that we need to understand how syllogisms are led back into the figures (46b40-47a2). In order to successfully complete this task, we must be alert to the possibility that 'both in writing and in conversation' premises can be omitted or unnecessarily stated.³⁹ Neglecting these possibilities and taking the premises of an argument at face value will make this task impossible (47a21-2). We must, then, investigate the 'inadequacy' (*to endees*, 47a22) of arguments that results through the aforementioned omissions and superfluous additions. Alexander dedicates multiple pages (341-9) to a detailed interpretation of Aristotle's examples, which, although interesting in itself, cannot be reproduced here in detail. It will suffice to point out a few crucial passages.

³⁹ I.32, 47a14-8. Aristotle in particular mentions the following cases: (ia) stating a universal premise but omitting to state the premise contained in it; (ib) stating a premise, but omitting the premises from which they are derived; (ii) asking for superfluous premises.

The problem Alexander detects in Aristotle's examples is the omission of premises. It is interesting to see what status these premises have. According to Alexander, we need to purge from arguments what is superfluous and add *what is necessary* until 'we reach and discover the two premises for the conclusion *in the strict sense* (*tas duo protaseis tas tou sumperasmatos kurias*, 343,31-2)'. Alexander, then, does not seem to accept that the arguments *as stated* are valid; rather, if we do not add what is necessary and hence reveal the premises in a strict sense (*tas kurias protaseis*) the argument is fallacious. That Alexander is not merely talking about s-invalidity becomes clear from two passages of his commentary: first, throughout the commentary of I.32 Alexander interprets *to endees* not as 'inadequacy with respect to syllogisms' but as 'fallacious argument' (i.e., invalid argument);⁴⁰ and secondly, explaining Aristotle's first example (destruction of substances) he says: 'the conclusion is not the result of the assumed sequence [of the argument], but [follows] because of the fact that the universal premise, which we added, is true [...]' (348,2-3).

But if Alexander is right and the arguments as stated are fallacious, why does Aristotle say that in them something follows necessarily from the assumptions? Alexander's answer is slightly different for the different cases he considers, but there is a common core in all of them: there is an omitted premise, which is true; and the addition of this premise makes the argument valid. Of course, not every argument that omits a premise would fall into the category of arguments that are not valid as stated, but nevertheless have a conclusion that follows, in this special way, 'necessarily from the assumptions'. The special case is rather characterised by the fact that the omitted premise is either a very obvious truth (as in the case of the axiom 'things equal to the

⁴⁰ Alexander understands *to endees* not merely as inadequacy with respect to syllogisms, but as 'error' or 'failure' of arguments in general. Glossing 47a22 he says: *rhaion to phōrasai tēn kata tous logous hamartian* (346,26).

same things will be equal to each other', 344,17), or is 'equal in power' (*isodunamos*) to the premise that would be required for the argument to be valid strictly speaking. It is not entirely clear what Alexander means by the term *isodunamos*, but the most important feature of premises that are *isodunamos* seems to be that they can be transformed (*metalambanesthai*) into or that they indicate the substitution of the required premise.⁴¹

Alexander proceeds to formulate a general explanation of the distinction between the two kinds of arguments and establishes a link to the definition of the syllogism. The word *tethentōn* (posited) in the definition should be understood as 'assumed to be or not be', i.e. stated explicitly in the way proper for syllogistic premises and not as 'hypothesized' (*hupotethentōn*), which, if we take into account Alexander's interpretation outline above, should be understood as 'somehow contained or indicated' by the premises or as 'covertly assumed' due to obviousness. Before trying to lead an argument back into one of the syllogistic figures, a process the chapter begins to investigate, one therefore has to find 'the conclusions' premises in the strict sense' (*tas kurias tou sumperasmatos protaseis*).

Therefore, Alexander does not read I.32 as an indication that Aristotle accepted a sort of logical consequence different from that of syllogistic consequence and hence as an acceptance of a validity different from s-validity. Although there are questions regarding the details of his interpretation (e.g., how to understand the term *isodunamos*), his general line of interpretation cannot easily be rejected. The idea that Aristotle is making a difference between arguments that are formally valid as stated and arguments that, although they are not valid in this sense, are still accepted as valid

⁴¹ Cf. Aristotle's discussion of enthymemes in II.27, where Aristotle explicitly says that in enthymemes some premises are not stated (*ou legousi*) while others are formally assumed (*lambanousin*), 70a19-20.

in an everyday context certainly has a foothold in Aristotle's characterisation of the latter arguments as 'in writing and in conversation' (47a16). Hence, even if Alexander's argument is not accepted as conclusive, the A-view can certainly not be established by reference to I.32.⁴²

Before we leave I.32, it is worth noting an interesting feature in Aristotle's choice of phrasing. Aristotle has several ways to signify logical consequence. The two most common ones in the *Prior Analytics* are *ex anagkēs sumbainei* ('it follows from necessity') and *anagkē esti* ('it is necessary'); a few times he also uses *sumbainei/ gignetai anagkaion* ('necessity follows/ comes about').⁴³ Aristotle's description of the non-syllogistic arguments in I.32, on the other hand, features a phrase that is unique in the *Prior Analytics*: *to anagkaion ti sumbainein*. With Alexander's interpretation in the background, it is very tempting to try and understand the *tis* as modifying *anagkaion* and create the same etiolated sense which Burnyeat (2002, p. 36-7) argued *paschein ti* and *alloiōsis tis* have in *De Anima* 410a25 and 416b33. The Greek of our passage does not allow for such a reading, as *tis* has a different grammatical function (in *De Anima* it modifies a noun, here it governs an adjective). Nevertheless, it is hard to deny that Aristotle's phrasing is much vaguer than if he had used his usual *ex anagkēs sumbainei* or *anagkē esti* – a vagueness that might well be thought of as indicative of the distinction Alexander draws between syllogistic arguments and the arguments discussed in I.32.

⁴² The idea that Aristotle's remark about necessary non-syllogistic arguments can be explained by distinguishing between 'necessary as stated' and 'necessary because of additionally assumed premises' can also be found in Striker (2009, pp. 214-5) and Crubellier (2014, p. 300).

⁴³ It is remarkable, however, that *sumbainei/ gignetai anagkaion* always seems to be used in sentences with negations signifying that the case under investigation does not allow for a logical consequence to be derived. Cf. 26a4-5; 26a7-8; 27a16-8; 27a25; 29a19-21.

Finally, I want to consider three passages, which, I think, contain evidence for the B-view and against the A-view. The first passage is part of the introductory paragraph of II.23:

We must now say that not only the dialectical and demonstrative syllogisms come about through the figures (*schēmatōn*) that we have described before, but also the rhetorical ones and simply any form of conviction whatsoever (*haplōs hētisoun pistis*), i.e. a conviction in accordance with whatever kind of inquiry. For everything we believe (*hapanta gar pisteuomen*) is either through syllogism or through induction. (68b9-14)

In order to have a correct grasp of the weight and scope of Aristotle's claim, we should first observe the context. II.23 begins the last section of the *Prior Analytics*, which continues all through to II.27 (cf. Smith 1989, p. 219). In this section, Aristotle attempts to put the technical theory of syllogisms, which he developed up to this point, in relation with different forms of creating convictions, for instance, induction and enthymemes. He argues that all these different forms of creating convictions rely on the technical syllogism. Enthymemes are a kind of rhetorical syllogism and they are characterised by the fact that a premise is missing. As such (i.e. *as stated*), the conclusion of an enthymeme is not logically necessary; but if the missing premise is in fact true and known, the enthymeme has a conviction-creating power and we might even say that the conclusion is a necessary consequence, because we assume the omitted premise. If we assume (*lambanousin*, 70a20) the omitted premise, i.e. if we disregard whether or not it is explicitly stated (*legousi*, 70a19), we end up with a technical and valid syllogism. Aristotle's theory of induction is a complicated matter

and cannot be discussed here in detail. II.23, however, clearly gives the impression that Aristotle thinks it also relies on technical syllogisms.⁴⁴

This leads Aristotle to make the very comprehensive claim that any conviction or belief comes about either through syllogism or through induction. It is clear that *syllogismos* here means ‘syllogism’ and not ‘argument’ (i.e. a more general notion comprising syllogisms and other valid arguments), for he explicitly links the word with the figures (*schēmata*). Moreover, if *syllogismos* would mean ‘argument’ it is difficult to see why he juxtaposes it with ‘induction’, if it is assumed that inductions are a kind of argument.⁴⁵ It appears to me, therefore, that the beginning of II.23 presents strong evidence for the view that Aristotle did not think of a kind of valid argument different from the syllogism and that he therefore equates s-validity with validity. The only way I see to make II.23 compatible with the A-view would be to understand Aristotle as saying that syllogisms and induction are only sufficient to bring about convictions of any kind but that arguments different from syllogisms might also be able to bring about some or all kinds of convictions. There is, however, nothing in the text that would suggest such a construal.

Secondly, one passage of Aristotle’s discussion of enthymemes might be of interest with respect to the question whether the A- or the B-view is more appropriate. I have already mentioned the fact that in this discussion Aristotle distinguishes between explicitly stating a premise and ‘not saying’ (*ou legein*) but assuming it. After these remarks, Aristotle makes a statement regarding the effect and status of an enthymeme as contrasted with a proper syllogism:

⁴⁴ 68b15-37. Cf. also McKirahan (1983).

⁴⁵ The difference between syllogism and induction is explained in 68b30-37. This explanation does not create problems for the claim that inductions rely on syllogisms, but it is difficult to make sense of it if *syllogismos* is understood as ‘argument’.

When only one premise is assumed, only a sign will come about (*sēmeion gignetai monon*), but when the other [premise] is also assumed, a *sullogismos* [will come about]. (70a24-5)

Aristotle contrasts *sēmeion* and *sullogismos* on the grounds of the number of premises explicitly stated: if both premises are stated, we will have a *sullogismos*; if only one premise is stated, we will only have a sign.⁴⁶ A sign, clearly, is weaker than a necessity and, hence, it seems justified to conclude that enthymemes in themselves (i.e. without additionally assuming the omitted premise) do not bring about a logical consequence and are strictly speaking invalid. More importantly, this passage is problematic for a claim that is sometimes made in connection with the A-view, namely that Aristotle regarded as valid one particular kind of non-syllogistic argument, i.e. arguments with only one premise (Frede 1974, p. 21). We have to ask ourselves, then, why a syllogism with one premise would not entail a logical consequence but only a sign while another kind of argument with one premise would bring about a logical consequence. If the A-view is spelled out such that Aristotle accepted a kind of propositional logic, there does not seem to be a plausible answer to this question, for a syllogistic premise is as good a proposition as any other.

Thirdly, and finally, a passage we have already discussed, I.23, can be referred to in favour of the B-view. Discussing the argument $AC \vdash AB$ (i.e. an argument with only one premise), Aristotle states that we need to assume another premise AX or CX or otherwise we will not have a syllogism. Explaining why this is the case he says:

‘for (*gar*) nothing follows necessarily (*ouden sumbainei ex anagkēs*) from the

⁴⁶ Aristotle contrasts *sēmeion* (sign or indication) with *sullogismos*. A *sullogismos* can, of course, be an argument; *sēmeion*, on the other hand, can hardly signify ‘argument’. But if we understand the words to signify the outcome of an argument and *sēmeion* is ‘sign’ or ‘indication’, *sullogismos* should be ‘consequence’. This is close to Smith’s suggestion to understand *sullogismos* as ‘deduction’.

assumption that something is said of something' (40b35-6). It is important to observe the precise chain of reasoning Aristotle presents us with. He first concludes that the single premise AC will not yield a syllogism. He then, as is regularly the case in Aristotle, provides the reason for the conclusion, namely that 'nothing will follow necessarily' from the assumption. The reason Aristotle provides is sufficient if we read it against the background of the definition of the syllogism: having a necessary consequence is a necessary condition of valid syllogisms and, hence, if an argument fails to satisfy this condition, we are entitled to conclude that this argument cannot be a valid syllogism. But what reasons are there to justify Aristotle's claim that nothing follows necessarily from the single premise AC? As I have argued above, there is a reason justifying this claim. This reason, however, is intrinsic to the theory of the syllogism and does not straightforwardly apply to logics different from the syllogistic. If Aristotle believed that the only kind of formally valid argument is the syllogism, his claim that nothing follows necessarily from a single premise AC can be sufficiently accounted for by the reasoning outlined in section 3. But if he believed that there is another kind of formally valid argument, then the claim that nothing follows necessarily from the single premise AC cannot be accounted for by constructing an argument based on statements Aristotle makes in the *Prior Analytics*. If the B-view is correct, Aristotle's argument at 40b35-6 is clear; under the A-view, however, it is not as clear how the argument should be understood.

To sum up: the evidence in favour of the A-view is certainly not conclusive. The most important passage that is supposed to corroborate this view, chapter I.32, can indeed be read in such a way. But, I think, it cannot be denied that Alexander's interpretation is also possible and that there are passages in Aristotle's text that

indicate such an interpretation. Moreover, Aristotle's aim to show that other kinds of arguments, such as enthymemes and inductions, ultimately rely on syllogisms or even have to be converted into syllogisms in order to be valid can be seen as strong evidence in favour of the B-view. To be sure, problems remain in Aristotle's syllogistic theory if we adopt the B-view (for instance, the status of conversions and hypothetical syllogisms). These problems might disappear if we adopt the A-view, but in the face of the textual evidence I do not see any justification for doing so. In my view, the fact alone that problems disappear from Aristotle's theory if we ascribe a notion to him which he never unequivocally embraces – let alone attempts to analyse – stretches the principle of charity too far.

I take the evidence for the B-view, then, to be more compelling. Moreover, given the textual situation, an adoption of the B-view is suggested by a principle of prudence with regard to historical texts. Even so, I do not think that the argument against the A-view and for the B-view is conclusive.⁴⁷ Hence, I will end on a conditional note: if my argument for the B-view and against the A-view has been convincing, nothing stands in the way of proceeding to the conclusion that Aristotle's syllogistic is a relevance logic: s-validity has been shown to have good approximations with the conditions laid out in modern relevance logic à la Anderson and Belnap and if the B-view is correct then s-validity is the only validity Aristotle accepts. Therefore, the conditions of s-validity are conditions of validity. If, on the other hand, the A-view is still seen as more convincing, this conclusion is not possible: the syllogistic satisfies the appropriate relevance conditions, but since these conditions are formulated as conditions of s-validity and modern relevance logic

⁴⁷ Given the nature of the A-view, which ascribes to Aristotle a notion of validity as an unanalysed primitive, and the Aristotelian texts we have, it is doubtful whether a conclusive argument against the A-view can be made at all. It is hard to argue against a view, when it is based in part on the idea that there is no textual evidence for what it claims.

demands them to be conditions of validity, Aristotle's syllogistic does not satisfy Anderson and Belnap's most fundamental requirement. Of course, it might still be possible that Aristotle thought that the more general notion of validity also satisfied relevance conditions. Yet, since he never analyses anything but s-validity, there is no way to investigate this thesis.

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